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Teaching electromagnetism in terms of potentials instead of
the "Maxwell" equations

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A recent survey [1] of electrical engineering courses in the UK has documented the reduction in the time allocated to the teaching of electromagnetics. This helps to highlight a major problem in that, although electromagnetism is of fundamental importance, much of the material which is traditionally taught is relatively little used by many, perhaps most, engineers. It can be omitted because the usual treatment of "field theory" reflects the view of all modern textbook authors that electromagnetism depends on the so-called "Maxwell" equations, expressed in terms of the vectors \mathbf{E} and \mathbf{B} , which are what students find difficult, and also unnecessary for many other parts of the engineering course, together with much subsequent professional work. They were not, however, the equations which Maxwell himself actually proposed [2,3], showing that alternative approaches are possible.

Numerical methods now provide the core to the teaching of field concepts, and these usually compute the electric potential ϕ and the vector \mathbf{A} , whilst \mathbf{E} and \mathbf{B} require a substantial amount of post-processing, contrary to the usual view of \mathbf{E} and \mathbf{B} as the entities defining what is meant by the "field", and ϕ and \mathbf{A} as mathematical auxiliaries. Maxwell endorsed the computational view in that he chose \mathbf{A} as the fundamental magnetic (or "electrokinetic") field quantity, and defined \mathbf{B} as its curl, thus avoiding the complexities of the vector-potential approach to \mathbf{A} , which he explicitly rejected in formulating his "general equations of the electromagnetic field". The "magnetic field" becomes the vector \mathbf{A} , which is what is computed numerically, and provides a much more direct picture of electromagnetic induction than does the concept of flux. ϕ can likewise be taken as the definition of what is meant by the "electric field", and is a measure of the potential energy, whilst \mathbf{A} measures the inductive, or "electrokinetic" energy. This is in direct contrast to the usual approach in terms of the \mathbf{E} vector

$$\mathbf{E} = -\text{grad } \phi - \partial \mathbf{A} / \partial t$$

which mixes up the capacitive and inductive properties. It is the mixing, together with problems of "flux cutting" and the like, which are the sources of the considerable difficulties and errors in treating practical examples of electromagnetic induction.

Some are considered in the paper. It is also shown that the usual objections to the treatment of ϕ and \mathbf{A} as physically meaningful, on grounds of the arbitrary choice of datum and gauge, can be reversed. When ϕ is defined in energy terms the datum provides information which is discarded in the customary approach via \mathbf{E} , and the Lorentz gauge likewise has direct practical implications.

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The paper summarises the possibilities, and advantages, of teaching electromagnetic theory in terms of the quantities which are computed numerically, and choosing the "field" equations accordingly. The principle reason for attaching so much importance to the customary "Maxwell" equations in \mathbf{E} and \mathbf{B} is their use in predicting a finite velocity of propagation, c , in empty space, but Maxwell's own derivation did not require, or include, the equation

$$\text{curl } \mathbf{E} = -\partial \mathbf{B} / \partial t$$

Moreover, despite the importance of the prediction at the time, it has long since been overtaken by subsequent events, particularly relativity theory, which assumes the finite energy propagation velocity as axiomatic, requiring (and allowing) no "explanation". Since the propagation delay is familiar to all modern students, and is one which they have no difficulty in accepting, there are clearly many advantages in assuming, rather than deducing, the velocity c . That is, to add to the "static" relationship the simple requirement that ϕ and \mathbf{A} are retarded in time. The electrokinetic property, \mathbf{A} , then follows as a direct consequence of the potential energy due to ϕ , by adopting a digital pulse approach [4]. The constant μ_0 is derived, not assumed, and a very direct description of energy radiation is obtained in terms of the effects of retardation on the capacitances and inductances of the sources. This helps to provide a simpler and clearer understanding of problems of electromagnetic compatibility, which are often obscured, rather than clarified, by the conventional approach in terms of the "Maxwell" equations, and the Poynting vector ExH .

As Feynman has pointed out, there are very fundamental reasons for adopting the ϕ, \mathbf{A} view, since it is these which appear in the Schrödinger equation, as the electromagnetic field quantities, not \mathbf{E} and \mathbf{B} . This is a matter of considerable importance because of the increasing role of quantum electrodynamics in electrical engineering. Moreover, the conventional view leads to apparent inconsistencies between the quantum and classical theories, shown by the Aharonov-Bohm effects. The ϕ, \mathbf{A} interpretation removes these anomalies [5], and also shows that the photon model of radiation is more closely related to the macroscopic ϕ, \mathbf{A} description than it is to the Poynting vector, ExH . An underlying advantage of the ϕ, \mathbf{A} treatment is that the usual \mathbf{E}, \mathbf{B} vectors are obtained merely by differentiation, so that the two points of view are directly linked, and the "numerical" model is seen to provide what is essentially a "short-cut", by using the various equations in a selective way. The importance, in teaching, is that we defer the treatment of the concepts and equations which require the most time, are the traditional source of difficulty, and are the least used by those engineers who do not require expertise in specialised applications.

- [1] Results of questionnaire on electromagnetics teaching in undergraduate course in electrical engineering in the UK. Conducted by the University of Bath, Dec. 1989
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- [3] Maxwell, J.C. "A dynamical theory of the electromagnetic field" Phil. Trans. R. Soc. 1865 155 pp 459-512 (various reprints, Inc. Scottish Academic Press 1982)
- [4] Carpenter, C.J. "Digital-pulse approach to electromagnetism" IEE Proc.A 1988 135 (7) pp 477-486
- [5] Carpenter, C.J. "Electromagnetic energy changes due to charges moving through constant, or zero, magnetic field" IEE Proc.A 1991 138(1) pp 55-70