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Teaching electromagnetism in terms of potentials instead of fields

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Summary

The advantages to the engineer of a radical change of approach to electromagnetism are outlined and discussed. Attention is directed away from the fields in empty spaces around the conductors to the source charges and currents inside them by redefining energy, momentum and power flow as properties of the charges. The potentials acquire a greater significance than the field vectors, giving a description which corresponds directly to the use of equivalent circuits. The Maxwell equations are removed from their central role and derived as auxiliary relationships, of interest primarily to specialists.

1) Introduction

The teaching of electromagnetism has long been in question, for a variety of reasons, including the pressure of material in undergraduate courses. One major factor is what is often seen to be the lack of relevance of the traditional material to the needs of the modern engineer. Electrical machines are given much less time than in the past, and the broadening range of interests in communications likewise reduces the emphasis on electromagnetic fields. Yet electrical engineering might well be defined as applied electromagnetism, and the need for some understanding of the subject can hardly be in doubt. The question is the extent and nature of that understanding in introductory courses intended for the broad range of engineers. The effect of modern trends is to highlight the differences between the practical experiences of most engineers and what is usually taught, which centres on field theory, as summarised by the "Maxwell" equations. The inability of many engineers to quote, or remember, these with any confidence illustrates their relative lack of relevance to their practical needs.

The paper summarises some of the possibilities of closing the gap by a radical change of approach, considered in more detail in papers listed. The treatment follows Maxwell's own description of electromagnetic induction, in his "general equations of the electromagnetic field". These were not the same as those which are now universally attributed to him, but were formulated in terms of the vector **A** as the base measure of magnetism, and the vector **B** as an auxiliary quantity defined as the differential of **A** (i.e. **curl A**). This removes the concept of flux from its central role, and is one of many examples of the reversal of conventional ideas. The potentials are used, in a self-consistent way, in place of the field vectors as the principle measures of the charge interactions. Another example of the reversal is the role of retardation. A finite propagation velocity, c, is assumed at the outset, and the value of c treated as something to be measured, instead of being derived in the customary manner as a consequence of interactions between electric and magnetic fields. The behaviour of pulses travelling along wires provides a simple way of reversing the logic and deriving magnetism as a consequence of retardation. It provides one example of the way in which the approach can be more closely related to modern needs, centred on digital and switching techniques.

What are known as the "Maxwell" equations lose their central role and are treated as auxiliary, of interest primarily to the specialist. Attention is directed instead to concepts which are directly illustrated by the use of equivalent circuits, and thus to the tools which are familiar to engineers more conversant with circuit theory than with field theory. "Field" solutions are now widely obtained by numerical packages, whose use usually depends on an understanding of potential functions. Treating these as the primary field measures redefines energy and momentum as properties of the charges inside the conductors, instead of the fields in the spaces around them. Power flow is described in a way which is familiar in terms of wattmeter measurements of voltage and current. This defines what is meant by "conducted power", and distinguishes it from other forms of emission, in contrast with the Poynting vector, **ExH**, which lumps them together. The distinction is fundamental to device design and to electromagnetic compatibility (EMC) requirements, which are of increasing concern in modern engineering.

2) Fields versus charges and potentials

The "general equations" which Maxwell set out in his Treatise caused his contemporaries considerable difficulties [1,2] for reasons which become clear in hindsight. His work embraced not one description of electromagnetism but two, and any understanding of them requires first that they are clearly separated. Their principal features can be shown most succintly by comparing the relevant energy-momentum 4-tensors, summarised informally in fig.1.

Perhaps the most important single point demonstrated by the tensor mathematics is the validity of each of the models as alternative descriptions of what is observed, since both are derived from the same expression for the electromagnetic forces acting on charges and currents. It is their behaviour which defines what is observable, not the "field" conditions which are derived from the observations. The "field" description of power transfer, for example, in terms of the Poynting vector, **ExH**, cannot be tested directly, and the need for inference is well recognised and documented, since there are an infinite number of other forms of Poynting vector, all leading to the same observable results.

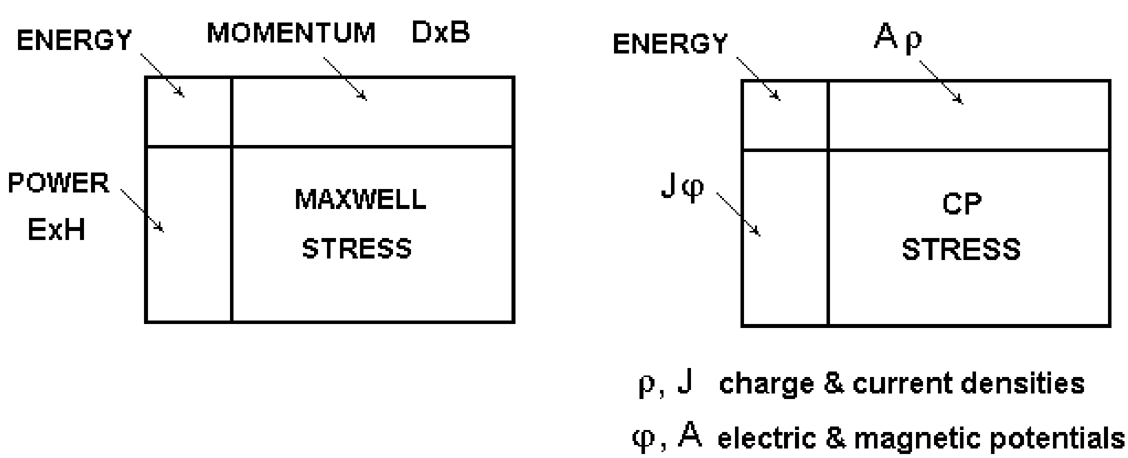


Fig.1 The two models
As summarised by the simplified energy-momentum 4-tensors (CP due to Endean {3})

It is appropriate to refer to the customary description of the electromagnetic properties of energy, power, momentum and stress as those of the "field" model, since all of the variables (such as **ExH**) are defined by the various products of the field vectors **E**,**D**,**H** and **B**, taken in pairs. In the alternative "charge-potential" model all of the properties are described instead as products of the charges or currents, of density ρ and **J** respectively, and the corresponding potential functions, one the electric scalar potential φ, and the other the magnetic vector potential **A**. The practical significance of the two alternatives is best illustrated by example, such as the "jumping ring", consisting, in essence, of a transformer with a straight laminated iron core whose secondary is conducting ring (fig.2). Energising the primary coil induces current in the ring and repels it. The device is a form of electrical machine whose action is described most directly as a repulsion between currents flowing in opposite directions, and this is also the most helpful view in that it shows the importance of time phase and the role of quadrature currents. However the early focus in introductory courses is on static devices, so that machines become a more specialist, and therefore deferred, topic. When the ring is held stationary then the mechanical force is of no interest, but the heating effect illustrates the emf induction which is central to the practical applications of electromagnetism for most engineers, whatever their particular expertise. The practical need may be to maximise the induced current, or increasingly to minimise it, as in EMC applications and the like.

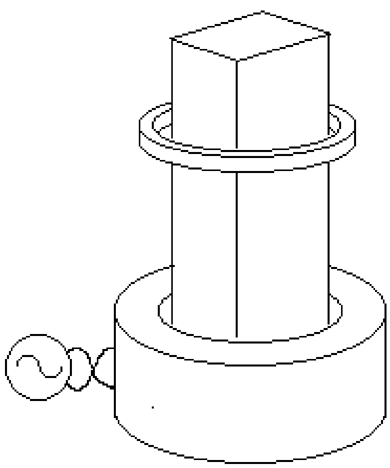


Fig.2 Jumping ring

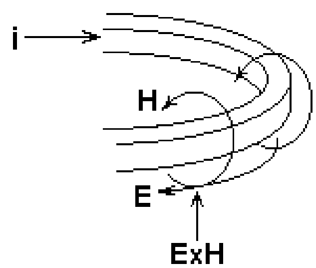


Fig.3 Poynting vector ExH

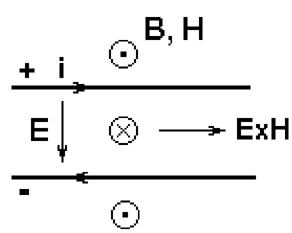


Fig.4 Supply lead ExH

The underlying requirement is the description of the transfer and conversion of energy. When the ring is held stationary the power conversion to heat is accounted for in the field model by the net **ExH** transfer through any surrounding surface (fig.3). The current, of density

$$\mathbf{J} = \sigma \mathbf{E} \tag{1}$$

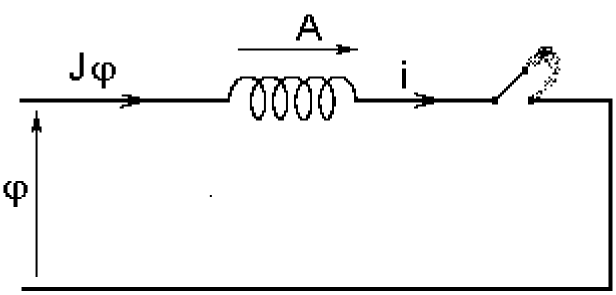
in a ring of conductivity σ produces an **H** field tangential to the surface, and requires a component of **E** in the direction of **J**, and an inwards component of **ExH**. But this power does not flow from the primary coil since, if the wire of the coil has zero resistance, then the axial component of **E**, and hence the normal component of **ExH**, is zero at its surface. A finite resistivity results in net **ExH** inwards, and this supplies only the loss, as in the ring. In this "field" view, it is not the primary coil but the supply connections which provide the energy input, as shown by the field conditions in the space around the wires, giving an **ExH** vector pointing in the parallel direction (fig.4). If the ring is replaced by a secondary coil then **ExH** likewise describes the transfer of power in the space around the output leads into the external load. The conductors "steer" this power flow, and **ExH** "drops off" enough to supply the losses, together with the stored energy. It does not otherwise penetrate into the interior spaces of the coils or windings, but appears as very large power densities around the end-connections when the power levels are high. The "field" description of power flow in the near-field devices is thus of little interest to designers, who pay relatively little attention to the end-connections, and **ExH** is ignored in virtually all of the relevant texts. Its practical application is limited to far-fields, such as those produced by wire antennas at remote points. The use of loop antennas emphasises the comparison with the transformer, and the inductive nature of the coupling. Again the near-field **ExH** is too complex to be of much practical interest.

Poynting's theorem replaced an earlier view of conduction as a process by which the charges themselves carry power inside the wires, instead of in the surrounding spaces. The charge-potential mathematics shows that the older view is not naive, or limited in its application. It supports a picture which is familiar to all engineers in the use of conventional forms of wattmeter to measure power, i v, in terms of the current i flowing in a wire at voltage v. **J**φ defines what is meant by the "conduction" power density in terms of the current density **J**, and φ provides a more exact definition of "voltage" in terms of what is often referred to as the "electrostatic" potential, although it is not confined to statics, but describes all of the electrical interactions due to excess charge. It demonstrates the need to connect the voltage coil of a standard wattmeter so as to measure φ, and exclude other sources of voltage, such as induced emfs. That is, to separate out effects which are usually associated with capacitance from those described as inductive, and this helps to illustrate the close link with ideas which are usually expressed in terms of equivalent circuits. The definition of what is meant by "conducted power", or "conducted emission", is directly relevant to EMC applications, in which the control of emissions depends on the recognition, and separation, of components which the Poynting vector, **ExH**, combines into a single term.

The prominence of **J**φ in fig.1 reflects the practical importance of conduction in all devices, including radio antennas, in which the necessary observations are not of **ExH** in empty space, but the power flow ij in some form of test antenna. This illustrates the distinction between φ and the induced voltage, and the need to add other terms to **J**φ to account for energy transfers across the empty spaces, as, for example, to a secondary antenna, or to the jumping ring. Since it is the emf induced in the ring which provides the energy, these points are closely related, but it is convenient to consider some of the other aspects of the field and charge-potential descriptions before coming back to power transfer.

3) Momentum

When the power supply to the primary coil is interrupted, by opening a switch (fig.5), the stored energy of the moving charges is translated into an arc, which may be destructive if the inductance is large and the coil is fed from a dc source. As Maxwell observed, the circuit behaves as if the "electrokinetic momentum" of the "electrical fluid" in the wire carries it across the gap. The momentum density, of charges of density ρ, is ρ**A**, a concept familiar to physists as "canonical momentum". It corresponds to the momentum, **DxB**, which carried by the field as a consequence of the magnetic energy (or, more directly, as a consequence of Maxwell stress in fig.1). The behaviour of the conduction electrons can be described in terms of **DxB**, just as the power transfer is equivalent to **ExH**, but the **DxB** analysis is relatively complex, and is not necessary to illustrate the point that ρ**A** provides the more direct description because it is the behaviour of the current within the wires (and the corresponding values of **A**), rather than the conditions in the surrounding space, which are of principle interest.



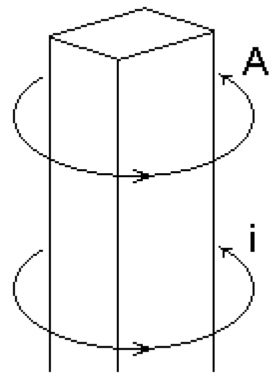


Fig.5 Momentum as measure of inductance **Fig.6 A vector due to coil**

The momentum q**A** associated with a charge q is characterised by the force which is caused by any changes in time. The force per unit charge

$$\mathbf{E} = - \partial \mathbf{A}/\partial t \qquad (2)$$

defines what is meant by "induced emf", per unit length of conductor. From equ.1,

$$J = - \sigma \, \partial \mathbf{A}/\partial t \qquad (3)$$

in the jumping ring, and the induced current provides a direct illustration of **A**. Since the interaction is between parallel currents, **A** points in the direction of the source current, as shown in fig.6 (see section 7), and provides a simpler description of the field than the flux density, **B**. Another example of its physical significance is shown by the effect of the momentum change of the "electrical fluid" at a bend, as in electromagnetic railguns (fig.7). The flow of current through the projectile carries the conduction electrons from the **A** vector at one rail into that of the other, and causes a corresponding change in ρ**A**. Equ.2 shows that the force on length δy of a part of cross-section s is

$$\begin{aligned} \delta F &= \rho \, \partial A_x/\partial t \, \delta y \, s \\ &= \rho \, u \, \partial A_x/\partial y \;\; \delta y \, s \\ &= J \, s \, \partial A_x/\partial y \;\; \delta y \qquad (4) \end{aligned}$$

where J is the current density due to the flow of charge at a mean drift velocity u. Integration along the armature between the ends 1 and 2 shows that the total force due to a current i is

$$F = i \, (A_{x1} - A_{x2}) \qquad (5)$$

where A_x is the component of **A** due to the current flowing in the rails from which the armature is fed.

Equ. 4 gives the same result as is obtained from the usual **JxB** expression for force, but the concept of the flux density **B** as a "cause" of the force is replaced by that of an action-at-a-distance effect between the moving charges, as described more directly by **A**. The change in view helps to remove various apparent anomalies which tend to surround simple devices of this type, and it also illustrates the point that the description of force which characterises the field model is the Maxwell stress. **JxB** is common to both models, as illustrated by equ.2, in which **B** denotes the differential of **A**. The equilibrium of the various parts of the system is accounted for in field terms by the Maxwell stress, which describes the recoil to F in terms of a "magnetic pressure" in the interior space, transferring the force from one end of the gun to the other. But this demands extremely high force densities in the empty spaces, paralleling the large **ExH** power densities. Concepts which are familiar at low field intensities are widely rejected as "unreal"when applied to high-current and high-power devices such as railguns, although the view of **ExH** is more ambivalent than that of forces acting on volume elements of empty space. The alternative description of F in terms of **JxB** does not help, since this confines the force on the rails to the direction normal to them, and cannot account for the recoil. Yet F is caused by the currents in the rails, particularly in the region close to the armature. An interaction which was first studied by Ampere (who postulated an axial component of force on current elements) is clarified by the charge-potential description, in which the "electrical fluid" properties replace the Maxwell field stress (fig.1). The result is best described by analogy with the behaviour of the fireman's hose, in which the properties of the fluid are clearly separated from those of the container, and the reaction to the force at a bend is accounted for in terms of the pressure (or stress) within the fluid.

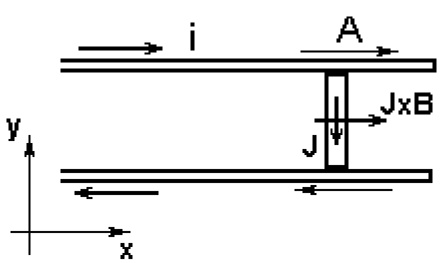


Fig.7 Momentum as measure of inductance

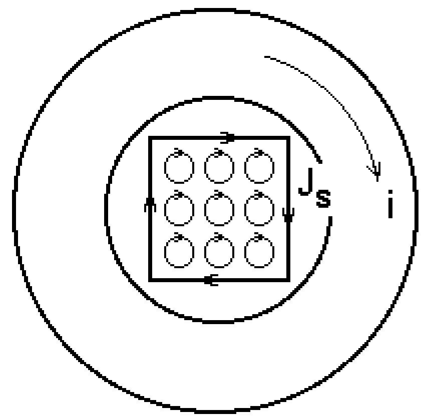


Fig.8 Ferromagnetism

The example provides a convenient illustration of the roles of the two models. Both are equally valid, and the differences are untestable, but the concept of pressures within conductor electron stream is likely to be more acceptable to engineers than that of a field pressure in the surrounding space, particularly when the forces are large. It helps to show more clearly why the action-reaction principle fails when expressed in terms of **JxB**, and it also illustrates an important underlying point. The charge-potential model is concerned only with the material volume elements of the system, and with the force and energy equilibrium of those elements. Because the underlying philosophy is of action-at-a-distance, no "mechanisms" are described, or needed, to transfer the forces and energies across the empty spaces, in direct contrast to the field model, which centres on those "mechanisms" (as shown in fig.1).

4) Induction and inductance

Mechanical forces are seldom of interest unless the parts move (by design or otherwise), although it may be remarked in passing that the prediction of the emf which is induced in the railgun can likewise be analysed more clearly in terms of **A** than **B**, since the armature moves through a flux field to which it contributes [4]. For the present purpose the focus of attention is on static devices, as illustrated by the induction in the jumping ring when it is held in a fixed position.

Changes in current with time produce a corresponding change in A, and an "electromotive force" on the conduction charges in accordance with equ.2. It is helpful to add the suffix **A**

$$\mathbf{E}_A = - \partial\mathbf{A}/\partial t \qquad (6)$$

to distinguish this from other forces on the same electrons, including what is commonly referred to as the "electrostatic" force, given by

$$\mathbf{E}_\varphi = - \mathbf{grad}\ \varphi \qquad (7)$$

The term "emf" is usually applied to the total voltage induced in any path

$$e = \int \mathbf{E}_A.\mathbf{dl} \qquad (8)$$

and this defines what is meant by "induction".

Since the potential **A** due to a current element is in the direction of the current flowing in the element (section 7), coiling up the wire of the primary coil likewise "coils up" the resulting **A** into circular paths concentric with the coil (fig.6). **A** induces the emf in the ring, and likewise in the coil, where it accounts for the coil inductance, L. That is, for the total momentum of the conduction electrons, due to the momentum density ρ**A**. L is usually defined in terms of the induced emf

$$e = - L\ di/dt \qquad (9)$$

so that

$$L = (1/i) \int \mathbf{A.dl} \qquad (10)$$

summed along the wire. Conversely, **A** is the inductance per unit length times the current.

Each of the electron spins in the ferromagnetic core is the equivalent of a small coil, and becomes a miniature "magnetic flywheel" because of its own ρ**A**. The total contribution to **A** is the same of that of a current $\mathbf{J}_s$ (fig.8) flowing along the surface of the iron, both in permanent magnets and in soft iron. The resulting increase in the momentum density, ρ**A**, of the conduction electrons accounts for the effect of the iron on the coil inductance. The charge-potential description thus provides helpful insights into various aspects of the system behaviour, as well as a convenient method of analysis, since L is more easily obtained from **A** than from **B**.

5) Maxwell force

The usual basis of the field description is the Lorentz force

$$\mathbf{F} = q(E + \mathbf{uxB}) \qquad (11)$$

on a charge q moving at velocity **u**. This is commonly used to define what is meant by the electric field, **E**, in terms of the force which acts on a stationary charge, and likewise **B** in terms of the component which varies linearly with velocity. Maxwell's approach was fundamentally different, in that his "general equation" recognised two sources of **E**. One is observed when q is placed between two charged plates, giving the force

$$\mathbf{F}_\varphi = q\ \mathbf{E}_\varphi = - q\ \mathbf{grad}\ \varphi \qquad (12)$$

and the other is illustrated by the jumping ring, in which the current is due to the F_A forces on the conduction electrons,

$$\mathbf{F}_A = q\ \mathbf{E}_A = - q\ \partial\mathbf{A}/\partial t \qquad (13)$$

Maxwell defined **B** as a symbol for the looping tendency of **A**, i.e.

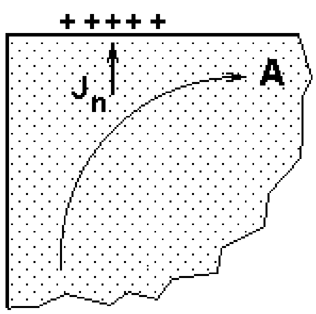
$$\mathbf{B} = \mathbf{curl\ A} \qquad (14)$$

so that his expression for force becomes

$$\mathbf{F} = q(-\ \mathbf{grad}\ \varphi -\ \partial\mathbf{A}/\partial t + \mathbf{uxcurl\ A}) \qquad (15)$$

A straightforward vector manipulation shows that the **uxB**, like the **E**, part of equ.15 can be separated into two components, and these also take a scalar (j-like) and non-scalar (**A**-like) forms, as is evident since we can transform to a reference frame in which q is stationary. The point has been discussed in ref.4. One practical advantage of the description of magnetism in terms of the $\partial\mathbf{A}/\partial t$ term, in place of **B**, in equ.15 is the displacement of **uxB** from the beginning to the end of a course which centres on static devices. The difficulties associated with the concept of "flux-cutting" are well illustrated by the extent of the literature discussing the range of apparent "anomalies" in the calculation of induced emfs [4].

The consequences of the replacement of the Lorentz **E**, in equ.11, by the Maxwellian **grad** φ and $\partial\mathbf{A}/\partial t$, in equ.15, are most easily illustrated by example. Replacing the jumping ring by a conducting path of any other shape, such a thin foil sheet with a central hole and a rectangular outer edge (fig.9), forces the current into a path which does not conform to the **A** map. If the foil is sufficiently thin then the induced current is "resistance limited", so that its contribution to **A** tends to zero, and can be ignored The $\partial\mathbf{A}/\partial t$ forces in the direction normal to the foil edge cause a charge "pile-up", and it is this which "steers" the current, in an entirely literal sense, into a tangential path, by creating the necessary."potential hill" in φ. The interaction forces are due to the excess charge, which is, in turn constrained by the surface energy barrier.



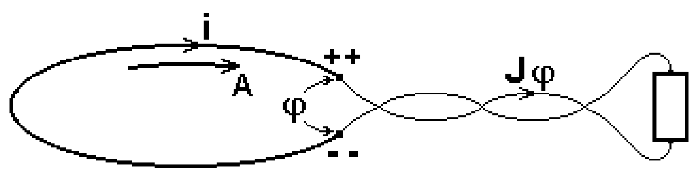


Fig.9 Current induced in foil sheet **Fig.10 Foil replaced by wire loop**

The change in **J** changes the heating effect by an amount which depends directly on φ and **J**φ. The conducted power is zero in a circular concentric ring, because the local heating is accounted for everywhere by the local induction, but otherwise the **grad** φ forces, and corresponding **J**φ, make an essential contribution in all problems of induction. Replacing the ring by a wire loop or coil, for example, as in fig.10, reduces the net **E** to zero in each turn if it carries no current, or if it has negligible resistance. The **gra**d φ and $\partial\mathbf{A}/\partial t$ forces are then equal but opposite. The resulting potential difference in φ across the terminals, due to the local build-up of charge, produces the **grad** φ forces needed to drive the current through an external circuit, in which the $\partial\mathbf{A}/\partial t$ forces due to the transformer are usually negligible if it is well-designed. Power is transferred "conductively" in **J**φ form to the external load, and it is only when there are no $\partial\mathbf{A}/\partial t$ forces in the leads that a conventional type of wattmeter can be used to measure the transfer. The user familiar only with the Lorentz **E** must beware. The **J**φ transfer is in direct contrast with the inductive coupling which conveys the power into the loop or coil, and depends on $\partial\mathbf{A}/\partial t$.

6) Equivalent circuit

The equivalent circuit restates the same ideas in graphical form. The sum of the $\partial \mathbf{A}/\partial t$ forces in the ring are represented, in fig.11, by what is shown as a secondary coil, and are balanced by the resistance drop when the coil is short-circuited. A more exact picture is obtained by distributing the resistance. In general the separation of excess charge in the primary coil represents stored energy, as represented by the capacitor, whose effect becomes important at high frequency. A more accurate model may then require a distributed capacitance to represent the energy distribution familiar in lumped form as a capacitance C, but more generally distributed over the conductor elements

$$C\,\varphi^2/2 = \int (\rho\varphi/2)\;dv \qquad (16)$$

The operation at sufficiently low frequency is characterised by the absence of any significant amount of capacitive energy, but this does not reduce the key role of the forces due to φ. Authors describing the same behaviour in terms of the Lorentz **E** tend to ignore the "electrostatic" contribution, leading some to abandon "fields" and turn to equivalent circuits, thus expressing in graphical form the underlying need for the charge-potential model.

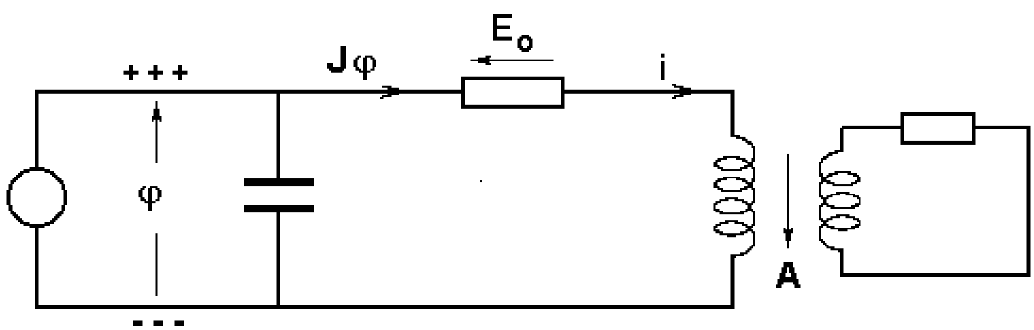


Fig.11 Equivalent circuit for "jumping ring"

The equivalent circuit illustrates the equilibrium condition, which can be expressed in the form

$$-\,\mathbf{grad}\,\varphi - \partial \mathbf{A}/\partial t + \mathbf{E}_o = 0 \qquad (17)$$

at all of the interior points of a stationary conductor. Integrating the current flow path between any two points 1 and 2 describes the balance in the three different sources of voltage

$$(\varphi_1 - \varphi_2) - (d/dt)\int_1^2 \mathbf{A.dl} + \int_1^2 \mathbf{E}_o\mathbf{.dl} = 0 \qquad (18)$$

The first defines what is meant by "terminal voltage" between 1 and 2, and the second the back-emf. The third describes the conversion to, or from, some "non-electromagnetic" form of energy, such as heat in a resistor, or chemical energy inside a chemical cell. Since all the forces on the charges are electromagnetic in origin, the definition of $\mathbf{E}_o$ as "non-magnetic" is essentially a matter of convenience, and the relevant energies provide the simplest way of identifying and separating the terms. When current is induced in a concentric ring, **grad** φ is zero, and the force balance is between the second two terms in equ.17, whereas in the magnetising coil the charge which is supplied by the power source raises the potential difference $\varphi_1 - \varphi_2$ to a level sufficient to drive the current against both of the other two terms, represented graphically by the coil and resistor symbols in fig.11.

The importance of equivalent circuits in the analysis of most electromagnetic device illustrates the practical need to replace the Lorentz **E** by its components. In stationary parts

$$\mathbf{E} + \mathbf{E}_o = 0 \qquad (19)$$

and illustrates the reverse of the usual interpretation of the electric field intensity, **E**. A resistanceless conductor imposes the local condition $\mathbf{E}_o$ =0, and hence **E**=0, but this does not imply the absence of an electromagnetic action. The $\partial\mathbf{A}/\partial t$ forces describe the long-range, or "electromagnetic" effects of the remote currents, and likewise grad φ describes those due to sources which are also remote, but are otherwise entirely different, in both nature and position. Using the symbol $\mathbf{E}_o$ emphasises the role of **-E** as a measure of the forces which are essentially local, and thus "non-electromagnetic", as defined by the relevant energy measures. A point which is obvious when expressed in equivalent circuit terms requires some emphasis because of the reversal of the usual "field" view. The force which acts on a test charge in the vicinity of a conductor is illustrated by the graphical symbol for resistance, and it is the mutual inductance which describes the elevtromagnetic action of the remote currents.

This assumes that the frequency is sufficiently low to make the induction entirely "magnetic", in the exact sense that the power transfers between the windings is accounted for by the axial forces due to $\partial\mathbf{A}/\partial t$. The conductive transfer of energy to and from the external sources and sinks depends entirely on the grad φ forces, but these contribute to the inter-winding transfers across the gaps only when the relevant capacitances are significant. Capacitance is defined by

$$C = Q/\varphi = (1/\varphi) \int \rho \, dv \qquad (20)$$

where Q is the source of the potential φ, as the parallel of the definition of inductance in terms of **A** (equ.10). The distinction between capacitive and inductive forces and energy transfers - i.e. between the two components of **E -** is at the heart of any understanding of device behaviour.

7) Systems definition of potentials

The emphasis on the Maxwellian components of the Lorentz **E** is in direct contrast to the customary use of **E** and **B** to define what is meant by the "electric" and "magnetic" fields, leading to the view, held very strongly by some writers, that the grad φ and $\partial\mathbf{A}/\partial t$ components of **E** are physically meaningless because they cannot be separated out by observing the force on a test charge. The usual derivation of φ from grad φ introduces what is seen as arbitrary constants of integration, and inverting equ. 14 likewise leaves the divergence of **A** undefined and apparently arbitrary.

The concept of "field" can, however, be approached very differently. The potential φ is defined uniquely, at any field point P at distance r from the origin, by the integral

$$\varphi = (1/4\pi\varepsilon_o) \int \rho/|r - r'| \, dv \qquad (21)$$

extended over all of the charges, of density ρ at point r', which together comprise the system. The "electric field" of a small group is described by the spherical set of equipotentials, varying inversely with distance, shown in fig.12, and that of other groups can be found by summation. The magnetic effect at P of any source charge, or group of charge, which moves at velocity **u** is given in terms of φ at P by

$$\mathbf{A} = u \, \varphi/c^2 \qquad (22)$$

That is, **A** is a vector pointing in the direction **u**, with a magnitude which varies in the same way as φ, whatever the size or shape of the group. Since current consists of charges of density ρ moving at a mean velocity **u** it follows that the system **A** is given by

$$\mathbf{A} = (1/4\pi\varepsilon_o c^2) \int \mathbf{J}/|r - r'| \; dv \tag{23}$$

The symbol μ_o is defined as

$$\mu_o = 1/\varepsilon_o c^2 \tag{24}$$

reversing the usual derivation of c from ε_o and μ_o.

In this view "magnetic" separates from "electric" in accordance with the velocity of the sources, not that of the test charge used to observe the Lorentz force. The test charge is fictitious, since its introduction changes the local **E** and **B** which it is being used to define, and it is the sources which are real and are the concern of the engineer. The use of charges and potentials can be categorised as a "systems" view, expressed in terms of potentials which are defined as properties of the system as a whole, in direct contrast to the customary treatment, which transfers attention away from the sources to points in space. The two views merely reflect different interpretations of the Lorentz force, both equally valid. Both are also equally familiar when the systems description is expressed in equivalent circuit terms, and the only major change is in extending this to volume elements.

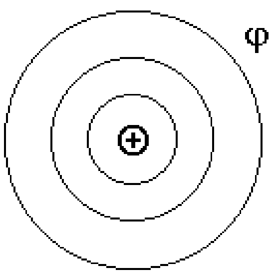


Fig.12 Electric field

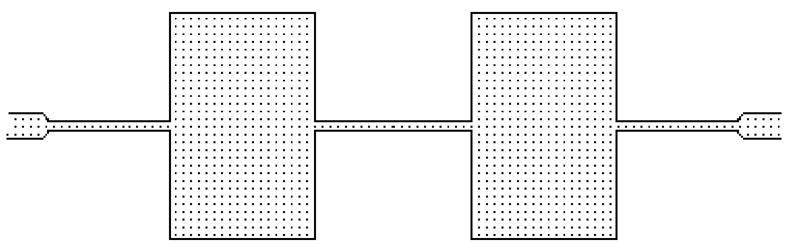


Fig.13 Microstrip filter

The sources ρ and **J** take time-retarded values, because of the assumption on which special relativity is founded. All energy transfers across empty space are necessarily limited to a finite velocity, independent of reference frame, whose value, c, has to be found by measurement. The retardation applies to both φ and **A**, in accordance with equ.22. Current continuity

$$\text{div } \mathbf{J} = - \partial\rho/\partial t \tag{25}$$

imposes the Lorentz gauge

$$\text{div } \mathbf{A} = - (1/c^2)\partial\varphi/\partial t \tag{26}$$

and this is automatically satisfied if the source condition is met. Assuming retardation thus defines φ and A at all frequencies, and the magnetic field of a current element is described much more simply and directly by **A** than in terms of the vector **B**, whose sources include both the current and the displacement current, ∂**D**/∂t. This is, of course, why **A** is commonly used in wire antenna applications. The underlying concepts are also simplified, since the limitations of space travel imposed by the velocity c are now sufficiently familiar to take for granted, whereas the idea of displacement current has long been regarded as one of the major conceptual hurdles of electromagnetism.

Other gauge conditions are often analytically convenient, but the Lorentz, or retarded, potentials retain their underlying role as the most direct measures of the "magnetic" effects of the remote currents, or moving charges, and the "electric" effects of the excess charges, as defined by φ. The φ=0 datum is not physically meaningless, but shows from where the charges have been separated [5]. This is a point which characterises "electromagnetism" as the explicit study of charge separation effects, in contrast with various other applications, such as "mechanical" stress, in which the electromagnetic effects are implicit. The practical siignifance of the φ=0 datum is illustrated by its central role in wiring and EMC regulations and the like. Other consequences of the use of φ as the principal measure of the "electric field" which have been considered in more detail in ref.5 include the treatment of polarisable materials and interfaces. The materials change the propagation velocity, but this does not invalidate the generality of the integral expression for φ and **A** (equ.21 and 23). As pointed out in the appendix to ref.5, any disturbance of the individual charges which constitute the material propagates at velocity c, but their response combine to change the resultant velocity, as demonstrated most easily by carrying terms from one side to the other in the corresponding differential equations. Amongst many other advantages of the scalar potential is the familiarity of voltmeters in making measurements, in direct contrast to the difficulty of observing **E** as the force on a test charge.

8) Long lines

Confining the current to long parallel conductors restricts **A** to a single component, and helps to illustrate the close relationship between the two potentials. A microstrip low-pass filter, acting as a transmission line over a ground-plane (fig.13), shows how the parameters of interest are described much more directly by the potentials than by **E** and **B**, and also the role of the capacitance. Approximating each section as part of a long line defines a capacitance per metre

$$C = q/\varphi \qquad (27)$$

where q is the charge per metre, and likewise an inductance per metre

$$L = A/i \qquad (28)$$

where A denotes the magnitude of the single component of **A**.

Since the sources are identical in form, equ.22 (or the comparison between equ.21 and 23) shows that the ratio

$$A/i = k\varphi/q \qquad (29)$$

where

$$k = \varepsilon_o\mu_o = 1/c^2 \qquad (30)$$

if the dielectric is assumed to have unit permittivity, for simplicity of illustration. Thus a high L/C ratio requires narrow strips giving large potential difference per unit source, and widening the section reduces the potential, with converse effects on L and C. Although this merely restates a familiar view, it shows the role of the potentials as the most relevant "field" measures, as well as the nature of the approximation in restricting **J**, and hence **A**, to one component.

The section impedance

$$Z = \sqrt{(L/C)} = \sqrt{(A/i)(\varphi/q)}$$

can be expressed in terms of either A/i or φ/q, as the same measure of geometry. Jφ describes the useful power transfer, and properly defines as "conductive" a device whose characteristics depend on large electric and magnetic field energies.

9) Magnetic flux

Describing induced emf in terms of **A**

$$e = -\ (d/dt) \int \mathbf{A.dl}$$

$$= -\ d\Psi/dt \qquad (31)$$

defines the symbol, Ψ, which is customarily interpreted as flux linkage. The relationship reduces to

$$\mathbf{curl\ E} = -\ \partial\mathbf{B}/\partial t = -\ \partial\ /\partial t)(\mathrm{curl}\ \mathbf{A}) \qquad (32)$$

when the path is closed, and sufficiently small, showing that the familiar flux-based picture is obtained by differentiation. But the popularity of this view tends to obscure the point that it is remarkably indirect, and that all of the practical considerations, both analytic and conceptual, support the use of **A** in place of **B**. Whereas the definition of flux linkage requires closed paths (leading some authors to claim that the concept of induced emf is valid only in closed circuits), equ.31 defines the emf for any part-circuit, such as a Hertz dipole. This is the most obvious of many other aspects of emf induction summarised below, and discussed at greater length in ref.4.

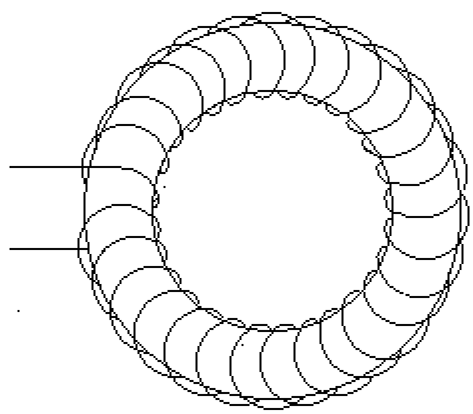


Fig.14 Toroidal winding

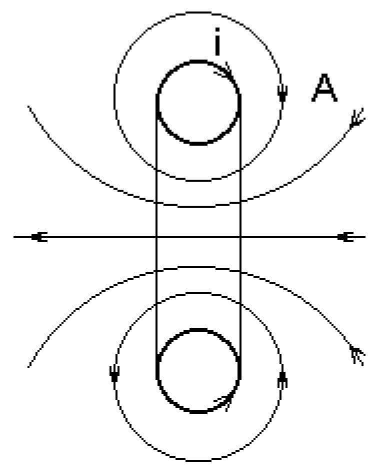


Fig.15 Toroid A map

A current-carrying coil of toroidal form (fig.14) produces an **A** vector which closes on itself, in both the interior and exterior spaces (fig.15), resulting in an approximately uniform curl **A** in the interior, but zero **B** outside. Because of the absence of any local curl **A**, no emf is induced in a small closed loop anywhere outside the toroid when the curent changes in time. But the exterior **A** map shows that emfs are induced in any external wire which is suitable oriented, and that this emf is the same for closed loops of any size which pass through the toroid opening. The induction, ∂**A**/∂t, force at any point in the wire depends on the local value of **A**, and "explaining" it in the customary way in terms of magnetic flux attributes an action-at-a-distance effect to **B**. The extent of the literature [4] discussing the implications of such ideas shows that the customary treatment of flux as the "cause" of induced emfs results in a far greater sense of mystery than that of action-at-a-distance by the source currents. It has probably accounted for more controversy and speculation than any other aspect of electromagnetism. A practical example is the application of the **uxB** to the calculation of emfs in conventional machines, in which the moving conductors are housed in slots, so that the surrounding iron tends to reduce the **B** field in the slot to zero if the conductor carries no current (fig.16). Yet this "screening" action has no effect on the induced emf, which is the same as if a wire taken out of the slot into the exterior **B** field in the air gap between the stationary and moving parts. Removing the arrows from the **B** lines provides a map of equipotentials in **A** (which has only one component if the field is two-dimensional, and is the quantity which is usually computed numerically). The emf is given by the local value of $\partial \mathbf{A}/\partial t$, and hence **A**, and the value of **B** in the vicinity of the conductor is irrelevant. The role of **B** has likewise caused much debate amongst physicists [6] in describing the magnetic effect of a field source such as the toroid on a charge moving past it. The change of quantum phase between electrons moving along the axis relative to others moving around an exterior path (fig.15) depends directly on the difference in the line integrals of **A** along the two paths, and thus provides a measure of **A**. The interaction is one which has attracted much attention [7] because it provides what is often presented as a unique example of a long-range action by magnetic flux in the absence of any local **B**,as well as having various practical applications. Perhaps the point which is illustrated most directly by such examples is the extent to which the field model, based on the concept of flux, has distorted and confused matters which would appear straightfoward if the potentials were more familiar as "field" measures.

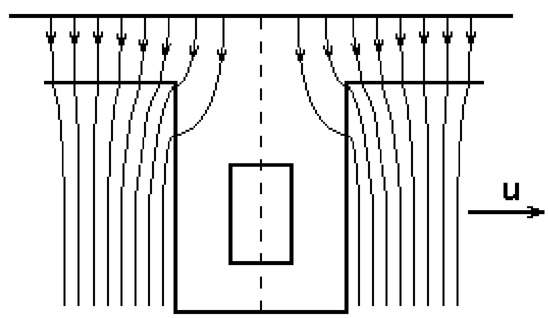


Fig.16 Conductor in slot

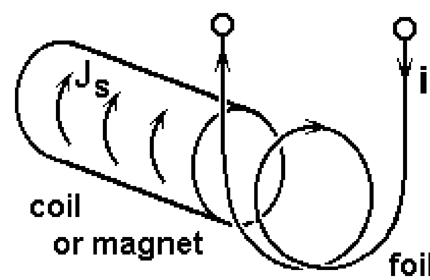


Fig.17 Foil response to magnetic field

10) Conducting foil

Removing flux as a base concept also removes the usual illustration in terms of iron filings, and suggests the need for some equivalent demonstration of **A**. The induction of current in the jumping ring provides one. Another is the behaviour of a thin current-carrying strip, cut from kitchen foil. This reacts to a magnetic field by tending to align itself in the direction parallel to **A**, and forms loops in the vicinity of a cylindrical coil, or magnet (fig.17). When the barrier limiting the motion towards the source is removed, the foils wraps itself around the winding, where **A** takes its maximum values. It reverses direction when the current is reversed. If the source is a magnet the foil traces out the paths of the equivalent surface currents, and shows how the magnet is magnetised. It thus helps to support the description of the field in terms of **A**, and demonstrates the close relationship with inductance, since the foil movement maximises the mutual inductance.

The behaviour of charges moving freely in a magnetic field, such as that due to the earth (fig.18) follows close parallels, better emphasised by turning the diagram through 90^o. Both examples illustrate the way in which the looping of the source currents tends to constrain the moving "test" charges into loops, and thus the practical significance of curl **A**. The role of **B** emerges, but in a way which demonstrates more directly the relationships between the two field measures and the source currents.

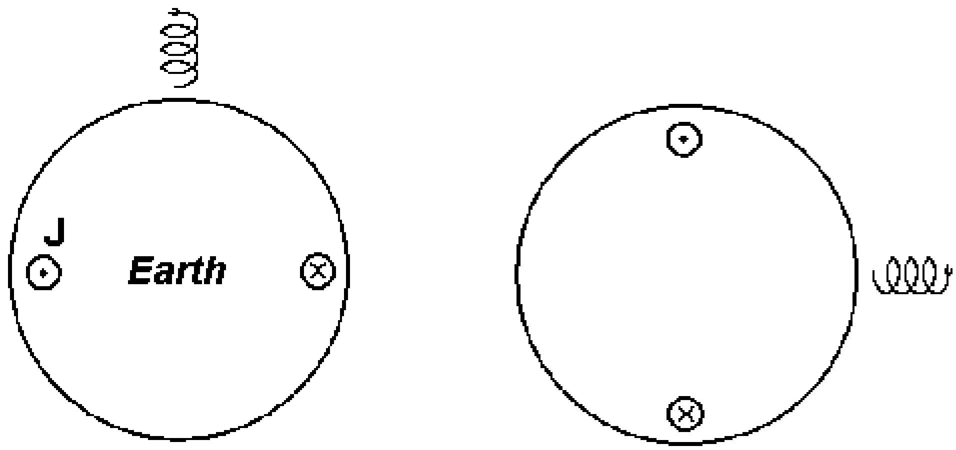


Fig.18 Charges moving through earth's field

11) Magnetism as a consequence of retardation

The charge-potential approach depends on retardation as an assumption, and provides a means of deriving magnetism from electrostatics in a way which is directly relevant to the emphasis on wavefronts and switching techniques in modern engineering. The time taken by a switching transient to travel along a pair of wires which are uniformly spaced and sufficiently long provides both a means of measuring c, and an example which is of immediate practical interest, following on an introduction to electrostatics in terms of φ.

The process of charging a line pair to a potential φ_o (fig.19) requires the supply of a charge, q_o, per unit length, and this necessarily takes a finite time since the total amount supplied tends to infinity. A change which is produced by closing a switch propagates as a wavefront, and this necessarily travels at the retardation velocity, c, of the electric potential, φ, in the absence of any other materials or sources of charge. Assuming steady-state conditions before the wave arrives requires uniformity of φ_o in the axial direction, and the same condition is imposed behind the wavefront by an energy source of constant potential. The power $i\varphi_o$ which is supplied by the source depends on the current

$$i = q_o\, c \qquad (33)$$

so that

$$i\, \varphi_o = q_o\, \varphi_o\, c \qquad (34)$$

This accounts for twice the electric (or "electrostatic") energy of the excess charge, q_o,

$$U_\varphi = q_o\, \varphi_o/2 \qquad (35)$$

per unit length, and the difference is the kinetic energy of the moving charges.

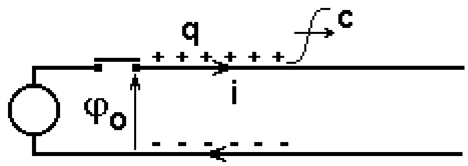


Fig 19 Line charging wavefront

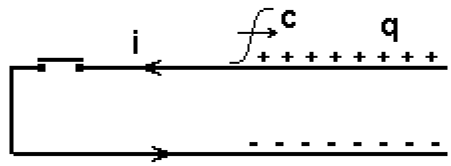


Fig.20 Line discharge wavefront

The energy balance is illustrated most simply by pre-charging the line to the potential φ_o, and reducing φ to zero behind the wavefront by imposing a sudden short-circuit (fig.20) . The current i then flows in the part of the wire in which $U_\varphi=0$, and has to be accounted for by the kinetic energy of the moving charges if energy is to be conserved. Since the balance does not depend on the conductor geometry, the kinetic (or magnetic) energy does, and thus requires a magnetic potential function, A, giving a kinetic energy

$$U_A = i\,A/2 \qquad (36)$$

per unit length, where the relationship between A and i must be geometrically similar to that between φ_o and q_o. Increasing the line spacing, for example, by an amount sufficient to double φ_o, with q_o constant, will likewise double A if i is constant. The condition

$$U_A = U_\varphi \qquad (37)$$

shows that

$$A = \varphi_o/c \qquad (38)$$

by substituting from equ.1. The unit of A is the volt-second per metre. The equivalence of magnetic and electric energy applies to any arrangement of parallel conductors, provided that they are sufficiently long. After the discharge wavefront arrives at any bend it acquires a component crossing through the conductor, generating reflections and new wavefronts travelling through regions in which the static charge distribution is not uniform.

The grad φ force which acts on a charge, δq, in the axial direction, z,

$$\delta q\ \partial\varphi/\partial z = -\ \delta q\ c\ \ \partial A/\partial z \qquad (39)$$

from equ.38, with a reversal of sign since the the reduction in φ corresponds to an increase in A in the discharge wavefront. Hence the equilibrium condition

$$\delta q\ \partial\varphi/\partial z + \delta q\ \partial A/\partial t = 0 \qquad (40)$$

in a resistanceless conductor, describing a momentum density ρA of charges of density ρ. It is this momentum which "piles up" the charge, and produces the corresponding "potential hill" in φ, when the current is interrupted by re-opening the switch. The current is due to the flow of conduction electrons, of density q_e per unit length, travelling with mean velocity u,

$$i = q_e\,u \qquad (41)$$

From equ.33

$$q_e = q_o\,c/u \qquad (42)$$

and the potential φ_e which these electrons would produces if separated from the lattice charges is

$$\varphi_e = \varphi_o\,c/u \qquad (43)$$

Substitution in equ. 38 gives equ.22, with the additional suffix e to distinguish the potential field of the moving charge, q_e, from that of the excess charge q_o.

The charge-potential treatment gives the pulse wavefront a dominant conceptual role, and encourages its early introduction in the undergraduate course, as appropriate to modern needs. The equivalent circuit provides the graphical equivalent, and is derived, not postulated. Some "field" texts likewise introduce line behaviour at an early stage, but have to assume a circuit model before demonstrating its validity in terms of the "Maxwell" field relationships later. The charge-potential approach to magnetism follows the underlying logic of modern physics in viewing magnetism as a relativistic consequence of electrostatics (fig.21). These ideas are customarily formulated in 4-tensor form in terms of the potential and current 4-vectors, and the subsequent derivation of the "Maxwell" equations relegates these to an essentially auxiliary role. The potential 4-vector components are in accordance with equ.38, and the source components follow from equ.33. Equ.22 provides the most succint statement of the relativistic nature of magnetism, and the pulse approach is inherently relativistic in deriving the kinetic effects as a direct consequence of a fixed propagation velocity. Both the Lorentz potentials, and the Lienard-Wiechert potentials for specific amouints of charge, are related by equ.22 under all conditions up to the velocity u<c, provided that φ and **A** are observed in the same reference frame.

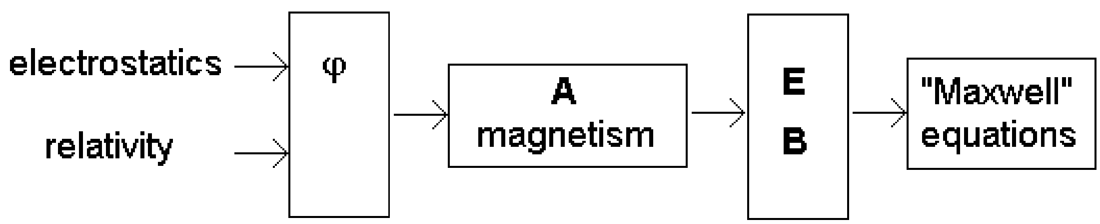


Fig.21 Electromagnetism based on relativity/retardation

12) Energy conversion

Jφ describes power transfer by conduction (i.e. "conducted emission") in any device or system, including the power supply to wire antennas and extraction from a receiver. It separates from the "capacitive emissions" and "inductive emissions" describing the transfer of energy across the empty spaces and through dielectrics or ferromagnetics. It is the remote transfers of power, due to the capacitive and inductive couplings, which characterise electromagnetism as the study of the effects of separating charges, or currents (fig.22).

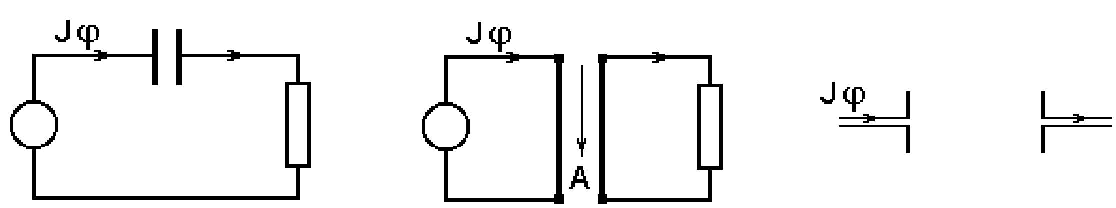


Fig.22 Power transfers

Before examining these transfers it is helpful to look first at the Poynting vector description, as set out in Poynting's theorem

$$\mathrm{div}(\mathbf{ExH}) + \mathbf{E}.\partial\mathbf{D}/\partial t + \mathbf{H}.\partial\mathbf{B}/\partial t - \mathbf{J}.\mathbf{E}_o = 0 \qquad (44)$$

for static devices. In a resistor

$$\mathrm{div}(\mathbf{ExH}) - \mathbf{J}.\mathbf{E}_o = 0 \qquad (45)$$

where $\mathbf{J}.\mathbf{E}_o$ describes the conversion to heat, as illustrated by the "jumping ring" when held stationary. The total loss in the conductor as a whole is accounted for, in field terms, by the transfer of power in through the surface at a rate **ExH**. When **B** varies linearly with **H**, then

$$\mathbf{H}.\partial\mathbf{B}/\partial t = (\partial/\partial t)(\mathbf{H}.\mathbf{B}/2)$$

and div(**ExH**) is accounted for in the usual way the rate-of-change of the magnetic field energy. But this does not apply inside a permanent magnet, where **B** does not vary linearly with **H**, and is not even in the same direction, giving negative **H.B**/2. This illustrates the wider significance of **H.**∂**B**/∂t, and **E.**∂**D**/∂t, like **J.E**$_o$, as measures of power conversion rates. The changes in the local field energy provide only a limited example.

The charge-potential equivalent

$$\text{div}\,(\mathbf{J}\varphi) + \varphi\,\partial\rho/\partial t + \mathbf{J.}\partial\mathbf{A}/\partial t - \mathbf{J.E}_o = 0 \qquad (46)$$

likewise describes energy conversion processes, the most obvious a resistor which is fed conductively. Equ.46 then reduces to two terms

$$\text{div}\,(\mathbf{J}\varphi) - \mathbf{J.E}_o = 0 \qquad (47)$$

one describing the conversion to thermal energy, and the other the balance by conversion from **J**φ. In general **E**$_o$ decribes conversions to, or from, any form which it is convenient to treat as "non-electromagnetic", reversing sign in a source supplying power. The closed ring is an example of current which is driven inductively by the induced emf , given by ∂**A**/∂, so that

$$\mathbf{J.}\partial\mathbf{A}/\partial t - \mathbf{J.E}_o = 0 \qquad (48)$$

Here the relationship

$$\mathbf{J} = -\,\sigma\,\partial\mathbf{A}/\partial t \qquad (49)$$

produces a 90^o phase difference between **A** and **J**, when the supply varies sinusoidally in time, and illustrates the generally non-linear relationship between the two. The interaction is typical of any coupled device in which the **A** vector at one coil includes a contribution from another. It is only in a single-winding inductor, in which the variation of **J** with time is everywhere the same, that the local **A** varies linearly with the local **J**, so that

$$\mathbf{J.}\partial\mathbf{A}/\partial t = (\partial/\partial t)(\mathbf{J.A}/2) \qquad (50)$$

This describes the conversion in a primary coil, of negligible resistance, when the secondary coil, or circuit, is removed. **J.A**/2 is the magnetic energy density stored in the conductor, which is a familiar alternative to the field equivalent **H.B**/2, and usually provides the base from which **H.B**/2 is derived. This is one of many examples of the familiarity of both models, and the somewhat ambiguous view of the charge-potential alternative, which is commonly seen as an approximate, equivalent, valid only under limited conditions (often not clearly specified).

The description of magnetic, or inductive, power conversion at a rate **J.**∂**A**/∂t is a concept which is very familiar to device designers as applied to the coil as a whole. The product of the current, i, and the induced emf, e,

i e = winding volt-amperes, VA,

usually measured in kVA or MVA for large devices, is obtained by summing **J** to obtain i, and ∂**A**/∂t to obtain e. **J.**∂**A**/∂t is the instantaneous volt-ampere rating per unit volume, describing the contribution to the total VA from each volume element. In practice the current usually varies sinusoidally in time, and the variables are defined by their RMS values, whereas **J.**∂**A**/∂t describes a local condition in both space and time. But the net result is the VA product, summarising the capability of the coil or winding as a magnetic device; that is, as a device which transfers energy magnetically. The "electric loading" of the material, **J**, is usually limited by heating, and ∂**A**/∂t depends on the frequency and the "magnetic loading", as observed by the conductor. The product defines the rate at which power is converted, giving a nett mean power transfer unless **A** varies linearly with the local **J**, when the energy is stored and recovered during a complete time cycle.

Closely coupled coils are those which tend to share the same **A**, under conditions in which the total currents in the coil cross-sections tend to be equal but opposite. Summing **J.**∂**A**/∂t through the section then provides a balance between the two energy conversions, one positive and the other negative in sign, and minimises the stored energy. Comparing this with the Ponting vector description illustrates the underlying differences between the two models. **ExH** provides a "mechanism" for the transfer of energy across the spaces around the conductors, and is largely concerned with energy conversions in those spaces, whereas the charge-potential description ignores the spaces, and is concerned only in the energy conversions within the materials, principally the conductors. No transfer "mechanism" across the gaps is provided, nor is necessary if we take the view that the engineer is interested only in predicting the observable behaviour. In general the materials include dielectrics and ferromagnetics, whose behaviour can be described in similar terms.

The $\varphi\,\partial\rho/\partial$ term in equ.46 likewise defines the process of capacitive, or "electric", energy conversion per unit volume, and distinguishes "capacitive" from "inductive" emissions. φ is commonly confined to short-range, or near-field, exchanges, and wire antennas are usually "magnetic" in operation, in the sense that the coupling with a remote antenna is described by **A**, not by φ. The practical value of such concepts is illustrated by the universal use of the volt-amp rating as the single most important measure of the capability of both capacitors and inductively-coupled devices, whereas the Poynting vector is usually found to have no useful application in near-field applications, and is almost completely ignored.

13) Conclusions

The charge-potential concepts depends on the clearer separation of two models, one based in the usual way on the description of energy, power, momentum and stress as properties of the field. Some aspects of the alternative are also familiar in the description of conduction as a process of power transfer by the conduction charges, and likewise magnetic energy density as **J.A**/2 instead of **H.B**/2. The corresponding canonical momentum ρ**A** is familiar to physicists. It has been shown that this alternative to the "field" model can be systematically developed to give a a self-consistent treatment, and removes the need to make Maxwell's equations the core of the course.

The alternative model offers many advantages, including:

1) the simplest description of induction and inductive coupling,
2) correspondence between "field" and "circuit" descriptions,
3) a definition of "conductive" emissions as Jφ and likewise of capacitive and inductive emissions,
4) magnetism founded on pulse behaviour
5) retardation becomes a base concept, and no displacement currents in space are required.

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15) Appendix

Equ.44, describing energy exchanges in static devices, can be derived by following Jackson [8[, for example, in analysing the energy exchanges, at a rate given by **J.E**, due to the flow of current. Jackson derives Poynting's theorem by starting with **J.E** and expressing **J** and **E** in terms of the field vectors. The alternative is to apply the Maxwell force to replace **E** by its components

$$\mathbf{E} = - \nabla\varphi - \partial\mathbf{A}/\partial t$$

so that

$$\mathbf{J.E} = \mathbf{J.}(-\nabla\varphi - \partial\mathbf{A}/\partial t) \qquad (51)$$

Expanding

$$\mathbf{J.}\nabla\varphi = \nabla\mathbf{.}(\mathbf{J}\varphi) - \nabla\ \nabla\mathbf{.J}$$

and substituting the current continuity condition

$$\nabla\mathbf{.J} = - \partial\rho/\partial t \qquad (52)$$

gives equ.46. Equ.52 is the only assumption, showing that the result is independent of gauge