A Critical Reexamination of the Electrostatic Aharonov-Bohm Effect

Allan Walstad
Department of Physics, University of Pittsburgh at Johnstown, Johnstown, PA 15904 USA
email: awalstad@pitt.edu

Abstract This paper undertakes a critical reexamination of the electrostatic version of the famous Aharonov-Bohm effect ("eAB"). The main conclusions are as follows: 1. Aharonov and Bohm's 1959 exposition is invalid because it does not consider the wavefunction of the entire system, including the source of electrostatic potential. 2. Although the authors attempted, in a 1961 paper, to demonstrate that consideration of the entire system would not change their result, they inadvertently assumed the desired outcome in their analysis. 3. An oft-cited observation claim cannot be accepted as such, because the experimental results are easily explained on the basis of electrostatic fields acting along the interfering particle paths, contrary to the defining premise of eAB. 4. An interference effect similar to eAB might be realized, but the magnitude of the effect would depend critically on the details of interactions within the experimental apparatus.

Keywords Aharonov-Bohm effect

1 Introduction

The Aharonov-Bohm effect refers to charged-particle quantum interference phenomena that can only be ascribed to the action of electromagnetic potentials, as the particles themselves propagate entirely through electric and magnetic field-free regions. Two versions were proposed in 1959 by Aharonov and Bohm [1], one mediated exclusively by the electrostatic potential (the electrostatic effect, called "eAB" in this paper), the other by the vector potential (the magnetic effect). The latter was soon observed experimentally [2,3], but confirmation of eAB proved more elusive. As this paper demonstrates, the theory underlying eAB suffers from a fundamental shortcoming that appears to have been overlooked for half a century. An oft-cited observation claim [4] is also shown to be mistaken.

Following Aharonov and Bohm (with modified notation), we recall that the Hamiltonian for a particle of charge q in a region of electrostatic potential V can be written $H = H_o + qV$, where H_o is the Hamiltonian in the absence of V. Schrodinger's equation for the Hamiltonian H is satisfied by

$$\psi = \psi_o e^{i\varphi}, \qquad \varphi = (q/\hbar) \int V(t)\, dt \tag{1}$$

where ψ_o satisfies Schrodinger's equation for H_o. The effect of V is thus to introduce a phase φ into the wavefunction.

If the wavepacket of a particle is split and allowed to propagate to a common destination via two different paths, and if V(t) is different on those two paths, then it seems the difference in phase should result in observable interference effects. Aharonov and Bohm described an idealized experiment in which the wavepackets travel through long, narrow metal cylinders, with V being turned on and off again for one cylinder only, while the wavepacket is safely inside (Fig. 1). They suggested that by changing the magnitude and duration of V, the resulting interference pattern would be shifted. Yet, the interiors of the cylinders are, to high accuracy, field-free regions. This proposed ability of an electrostatic potential to cause a physical effect, in the absence of an electric field or force acting on the particle, was counter-intuitive for many physicists at the time.

Given that the electrostatic and vector potentials V and **A** are Lorentz frame-dependent components of a relativistic four-vector, Aharonov and Bohm noted that the expression for electromagnetic phase must be generalized as follows:

$$\varphi = (q/\hbar) \left[\int V\, dt - \int \mathbf{A} \cdot d\mathbf{r} \right]. \tag{2}$$

Outside a long, thin solenoid, there is an azimuthal vector potential but no appreciable magnetic field. Quantum interference between trajectories passing on opposite sides of the solenoid will depend on the phase shift $(q/\hbar) \int \mathbf{A} \cdot d\mathbf{r} = (q/\hbar)\, \Phi$, where Φ is the magnetic flux through the solenoid. That is the magnetic Aharonov-Bohm effect. It is not the subject of this paper.

2 The Problem

As Sakurai points out in a well-known text [4], eAB is nothing other than an example of the energy-frequency relation $E = h\nu$. The energy of a charged particle depends on the electrostatic potential, and the rate of phase generated depends on the energy. If we can arrange for the voltage to differ along different interfering paths, then the phase difference and resulting interference pattern should depend on the voltage difference, even if the particle is not subject to an electric force on either path.

But this interpretation immediately raises a problem: conservation of energy requires that a change in the energy of the particle be matched by an equal and opposite change in the energy of the rest of the system. Quantum interference occurs between different paths of the entire system from one point to another in configuration space. If the particle's energy changes by ΔE for a time t, the extra phase for the particle is $\Delta E\, t/\hbar$. The energy of the rest of the system changes by $-\Delta E$ for the same time t, giving a phase of $-\Delta E\, t/\hbar$. The overall phase is unchanged.

To put it more formally, the amplitude characterizing the evolution of the overall system will consist of terms of the form $\psi = \psi^p \psi^s$ where ψ^p is the amplitude for the particle and ψ^s is the amplitude for the rest of the system. Consider two interfering paths by which the charged particle may travel from point *a* to point *b* via long metal cylinders as shown in Fig. 1. Along path 1, the potential is changed by V volts for a time t while the particle is well within the cylinder, changing the energy of the particle by $\Delta E = qV$. This gives $\psi^p = \psi^p_o\, e^{i\Delta E\, t/\hbar}$, where ψ^p_o is the amplitude for the particle if no voltage were applied. Then we must have $\psi^s = \psi^s_o\, e^{-i\Delta E\, t/\hbar}$, where ψ^s_o is the amplitude for the rest of the system if no voltage were applied. The overall amplitude for path 1 is

$$\psi = \psi^p\, \psi^s = [\psi^p_o\, e^{i\Delta E\, t/\hbar}]\, [\psi^s_o\, e^{-i\Delta E\, t/\hbar}] = \psi^p_o\, \psi^s_o. \tag{3}$$

Therefore, it appears that the application of a voltage to the cylinder does not alter the overall phase for that path and has no effect on the result of interference with path 2.

There is much more to the story, as we shall see later. But first: Just how is the energy of the rest of the system affected by the presence or absence of q? In order to change the voltage of a cylinder, charge is brought to its surface from elsewhere. Even if no net force acts on q, nevertheless q exerts forces on the approaching charges, thereby changing the amount of work required to charge the cylinder. This work comes at the expense of an energy source which must be considered part of the overall system for which quantum interference occurs.

One might wonder whether an analogous argument involving conservation of momentum should not apply to the magnetic Aharonov-Bohm effect. The phase gradient $(q/\hbar)\,\mathbf{A}$ is to be understood in terms of an electromagnetic momentum acquired by a charged particle in the presence of a vector potential. Conservation of momentum requires that an equal and opposite momentum and phase gradient be acquired by the rest of the system. But the mere existence of a phase gradient does not generate a phase unless there is an accompanying velocity; indeed, that is the significance of the $q\mathbf{v}\cdot\mathbf{A}$ term in the Lagrangian for a charged particle. Unless the rest of the system (or parts thereof) is moving with a velocity comparable to that of the particle, there is nothing to compensate the electromagnetic phase difference between the interfering paths in the magnetic effect.

The phase associated with electrostatic energy, however, comes about via a time duration, and the same time duration applies to both the particle and the experimental apparatus, at least as observed in any given Lorentz frame. That is why the equal and opposite energy shift generates equal and opposite phase.

3 eAB Observed?

Van Oudenaarden et al. [5] observed oscillations in the conductance of a mesoscopic metal ring as a function both of the applied voltage and of an externally-imposed magnetic flux through the ring. They interpreted the results as being due to a combined magnetic and electrostatic Aharonov-Bohm effect. Let us critically examine their experiment and its interpretation.

As illustrated in Fig. 2 (adapted from [5]), the metal ring is interrupted on opposite sides by tunnel junctions. Current flow from source to drain in response to an applied voltage V requires that electrons tunnel across these junctions. Tunneling results in an electron and a hole propagating in opposite directions from the junction. If the electron and hole propagate to the drain and source, respectively, they contribute to the current. If, however, the electron and hole travel around the ring and recombine at the other junction, they do not contribute to the current. Although the ring is not superconducting, the temperature is low enough that phase coherence is maintained despite the diffusive motion of the electron and hole. Quantum interference can therefore affect the likelihood that a tunneling event will contribute to the current, and thereby it can affect the conductance of the ring.

Consider the following two paths of the system between the same starting and ending state:

a) There is no tunneling event.
b) There is a tunneling event at one junction, after which the electron and hole travel in opposite directions around the ring and recombine at the other junction.

If the phase difference between these paths is 2π or a whole multiple thereof, interference will reinforce a result that does not contribute to the current, and conductance will be suppressed. If the phase difference is π or an odd multiple thereof, then this result will suffer cancellation and conductance will be enhanced. At constant V, conductance maxima are observed to occur at magnetic flux values differing by $\Delta\Phi = h/e$ (where e is the charge quantum), which is unambiguously what we expect from the magnetic Aharonov-Bohm effect.

The oscillations of conductance with magnetic flux are modulated periodically as a function of V. This dependence on V is interpreted by [5] in terms of eAB. On this interpretation, a phase $eV\, t_o / \hbar$ is generated during the time t_o between tunneling and recombination, while the electron and hole traverse their respective sides of the ring at a potential difference V. Setting $e\, \Delta V\, t_o / \hbar = 2\pi$, where ΔV is the change in V between conductance peaks, we can infer t_o.

The relationship between t_o and ΔV is correct, but it could not be an example of eAB. It is the defining premise of eAB that interfering particles travel through field-free regions, but in the experiment of [5] the electron and hole are subject to an electric field in tunneling across a junction. It is easily seen that the momentum thereby acquired is responsible for the interference, as follows.

Let s be the total distance traveled by an electron or hole in its diffusive motion from one junction to the other. At speed v the time required is $t_o = s/v$ and the deBroglie wavelength is $\lambda = h/mv$. The phase generated by mechanical momentum is $\varphi = 2\pi s/\lambda = 2\pi smv/h$. A change ΔV in the voltage causes a (small) change $\Delta v = e\, \Delta V / mv$ in speed, leading to a phase shift $\Delta\varphi = (2\pi sm/h)(e\, \Delta V / mv) = e\, \Delta V\, t_o / \hbar$. (This result is not altered by the fact that the energy eV is shared by the electron and hole.) Thus, the modulation of the conductance with changing V is due to ordinary electric forces. It is not an example of eAB.

4 Aharonov and Bohm's 1961 Paper and the Wavefunction of the Entire System

In the abstract to a 1961 paper [6], Aharonov and Bohm stated, "We…extend our treatment to include the sources of potentials quantum-mechanically, and we show that when this is done, the same results are obtained as those of our first paper…." For our purposes, the relevant part is their section 3. There, the authors wish to demonstrate that with suitable simplifying assumptions the overall wavefunction factors into a wavefunction for the source (having no dependence on the position of the electron) times a single-particle electron wavefunction obeying Schrodinger's equation with the time-dependent electrostatic potential V. This would give the same phase difference for the two paths of the electron as they derived in 1959.

Their eq. 11 is Schrodinger's equation for the entire system,

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x},\ldots y_i\ldots,t)=[H_e+H_S+V(\mathbf{x},\ldots y_i\ldots)]\Psi(\mathbf{x},\ldots y_i\ldots,t) \tag{11AB}$$

with **x** the location of the electron and y_i the coordinates characterizing the parts of the apparatus (the source of potential). We see that the Hamiltonian has been broken into three parts: one each for the electron and the apparatus, and an interaction potential energy V. (Note: Elsewhere in this paper V is used to denote electric potential, not potential energy.) They seek to treat the apparatus via a WKB approximation, writing its wavefunction (in their eq. 13) in terms of a magnitude R and phase S/$\hbar$ as

$$\phi(\ldots y_i\ldots)=R(\ldots y_i\ldots,t)e^{iS(\ldots y_i\ldots,t)/\hbar}. \tag{13AB}$$

S and R are said to obey, respectively, their eq. 14 and eq. 16:

$$\frac{\partial S}{\partial t}+\sum_i\frac{1}{2M_i}\left(\frac{\partial S}{\partial y_i}\right)^2+W(\ldots y_i\ldots)=0 \tag{14AB}$$

$$\frac{\partial P}{\partial t}+\sum_i\frac{\partial}{\partial y_i}\left(\frac{p_i}{M_i}P\right)=0 \tag{16AB}$$

where in eq. 16AB the substitution $P = R^2$ was made. [In eq. 14AB an obvious typographical error in the original paper has been corrected by replacing V(…y_i…) with W(…y_i…), the potential energy of interaction of all parts of the source with each other, introduced in eq. 12 of the original paper.]

The problem with eqs. 13AB, 14AB and 16AB is that they do not allow for any possibility of the electron affecting the wavefunction of the apparatus, as the position of the electron does not appear in those equations. Instead of demonstrating that the overall wavefunction factors as desired, Aharonov and Bohm in effect already assumed that because the apparatus is massive, it goes about its way as though the electron were absent. Their analysis breaks down at this point. The change in motion of the (parts of the) apparatus due to forces exerted by the electron may indeed be tiny, but only a tiny change in motion is required in order to effect a phase change of order unity, sufficient to invalidate the eAB result.

What remains here is to offer a correct analysis. We begin with the fact that interference occurs at points in the configuration space of the entire system. Thus, in any attempted experimental realization of eAB, no fact about the final state of the apparatus (or the environment) can be permitted to depend on which trajectory the electron followed. Otherwise, even though the electron paths meet at a point in space, that point corresponds to two different points in configuration space.

In the idealized experiment mentioned earlier (from Aharonov and Bohm's 1959 paper), suppose one cylinder is charged to a potential V from one plate of a very large capacitor. After a time, the charge is allowed to flow to ground. All of this happens while the electron is in the interference apparatus. To achieve a given voltage requires a different amount of charge if the electron is present in this cylinder than if it is not. (If we imagine spheres instead of cylinders, with the electron residing in the center of the sphere, then it is easy to see that the difference will be 1e of charge.) So the net amount of charge drawn from the capacitor depends on the path taken by the electron, and interference cannot occur. Suppose instead that we arrange for a precise, specific charge to be delivered to the cylinder; in that case, a different amount of work must be done to deliver the charge (due to the force exerted by the electron), depending on the electron path; this work shows up on the energy "balance sheet" of the source and again there can be no interference between the electron paths.

Nevertheless, the conditions for interference do appear to be met in another idealized experiment offered by Aharonov and Bohm, this one in section 3 of their 1961 paper. As illustrated in our Fig. 3, the interfering electron paths go around the outside of a charged capacitor, one on the positive side, the other on the negative side. The capacitor plates start at zero separation, so that there is no potential difference. While the electron is traveling by, the plates are given an impulse outward away from each other and then allowed to move freely under the influence of their electrical attraction. The plates reach a maximum separation and fall back together again before the electron clears the region. The electron feels no force, because the permitted paths remain outside the region between the plates. Nevertheless, there is a (time-varying) potential difference between the interfering paths, which, integrated over time during the period of plate separation (and multiplied times the electron charge over $\hbar$), would give a phase difference according to eAB. Let us examine this case in detail, first as an example of eAB and then with a correct analysis that treats the system as a whole.

For simplicity, replace the electron by a generic particle of positive charge q. Let the plates have charges +Q and –Q and area A. Let the negative plate have mass M while the positive plate is very massive so that its motion may be neglected by comparison. Let M be given an initial outward velocity v_o. The velocity v will decrease from v_o to 0 at maximum separation and reach $-v_o$ when the plate returns. The potential difference [positive side minus negative side] is $V = Qy/\varepsilon_o A$ where y is the plate separation as a function of time. The motion of the negative plate is a simple case of constant acceleration $a = -Q^2/2\varepsilon_o AM$, wherein we also have $y = (v^2-v_o^2)/2a$. We may also use the substitution $dt = dv/a$.

Putting these pieces together we have

$$\Delta\varphi = (q/\hbar) \int V\, dt = - (q/\hbar)\, (2M^2\varepsilon_o A/Q^3) \int (v_o^2-v^2)\, dv. \tag{4}$$

Evaluating the integral from v_o to $-v_o$, we find

$$\Delta\varphi = \varphi(+\text{side}) - \varphi(-\text{side}) = +(8/3)M^2\varepsilon_o A v_o^3 q/Q^3\hbar \tag{5}$$

as the eAB result for the phase shift.

A correct analysis of this experiment begins with the recognition that the rate of phase generation along a path in configuration space is the value of the Lagrangian of the entire system divided by $\hbar$ (give or take an overall minus sign, depending on whether one's preference in plane waves is $e^{+iEt/\hbar-i\mathbf{p}.\mathbf{r}/\hbar}$ or $e^{-iEt/\hbar+i\mathbf{p}.\mathbf{r}/\hbar}$). In the absence of any appreciable vector potential, the Lagrangian is potential minus kinetic energy; along the actual trajectory of the system in configuration space, that is the same as the total energy minus twice the kinetic energy, or twice the potential energy minus the total energy. Symbolically, we have

$$\hbar d\varphi/dt = L = U - K = E_{tot} - 2K = 2U - E_{tot}. \tag{6}$$

Although total energy is conserved, the work done by the electron's force on the parts of the apparatus changes the Lagrangian (and thereby the phase) through the kinetic energy.

The phase difference $\Delta\varphi$ that we are looking for is the difference in the integral

$$\varphi = (1/\hbar) \int L\, dt = (1/\hbar) \int (E_{tot} - 2K)\, dt \tag{7}$$

corresponding to the two electron paths. Since total energy is conserved, $\int E_{tot}\, dt$ does not depend on the path taken by the electron, and we only need find the difference in the quantity

$$\varphi = -(1/\hbar) \int 2K\, dt, \tag{8}$$

where the kinetic energy of the system (aside from the constant kinetic energy of the particle q) is just $K = (1/2)Mv^2$. In the absence of q, we have

$$\varphi = - (1/\hbar) \int M\, v^2\, dt = + (1/\hbar)\, (2M^2\varepsilon_o A/Q^2) \int v^2\, dv. \tag{9}$$

Evaluating the integral from v_o to $-v_o$, we get

$$\varphi = - (4/3)M^2\varepsilon_o A v_o^3/Q^2\hbar. \tag{10}$$

What is the effect on φ of the passing charged particle? This appears straightforward. If q goes by on the positive side, Q^2 gets replaced by $Q(Q+q)$ to account for the additional force of q on the negative plate. If q goes by on the negative side, Q^2 gets replaced by $Q(Q-q)$. Given that Q is much larger than q, the phase difference seems to be

$$\Delta\varphi = \varphi(+\text{side}) - \varphi(-\text{side}) = + (8/3)M^2\varepsilon_o A v_o^3 q/Q^3\hbar \tag{11}$$

between the two cases. So it looks as though Aharonov and Bohm obtained the right expression after all!

But not quite. Earlier we said that conservation of energy permits us to ignore E_{tot} in the Lagrangian $L = E_{tot} - 2K$, because $\int E_{tot}\, dt$ *for any given time period* does not depend on the path of q. Nevertheless, the time period between the initial impulse and the return of the plates together *does* depend on the path of q. The attraction of (+)q for the negative plate decreases the time of flight when q is on the positive side and increases it when q is on the negative side. By how much?

In the absence of q we have $t = -2v_o/a = 4\varepsilon_o AMv_o/Q^2$. And again, if q goes by on the positive side, Q^2 gets replaced by $Q(Q+q)$, whereas if q goes by on the negative side, Q^2 gets replaced by $Q(Q-q)$. In this way we see

$$\Delta t = t(+\text{side}) - t(-\text{side}) = -8\varepsilon_o AMv_o q/Q^3. \tag{12}$$

So when q passes on the positive side, there is an extra period $|\Delta t| = 8\varepsilon_o AMv_o q/Q^3$ to be accounted for, when the plate trajectory is complete but the plate trajectory for the other interfering particle path would not have been complete.

Suppose the kinetic energy of the returning plate, $K_o = Mv_o^2/2$, is dissipated entirely as random kinetic energy, as in an ideal gas; that kinetic energy is still in the system during $|\Delta t|$. So $\Delta\varphi$ gets an extra contribution of

$$\delta\varphi = (1/\hbar)(-2K_o)|\Delta t| = (1/\hbar)(-Mv_o^2)(8\varepsilon_o AMv_o q/Q^3) = -8\, M^2\varepsilon_o Av_o^3 q/Q^3/\hbar. \tag{13}$$

Then the total result for $\Delta\varphi$ is

$$\Delta\varphi = +(8/3)M^2\varepsilon_o Av_o^3 q/Q^3\hbar + (-8M^2\varepsilon_o Av_o^3 q/Q^3/\hbar) = -(16/3)M^2\varepsilon_o Av_o^3 q/Q^3\hbar. \tag{14}$$

So it is not only twice as large, but also in the opposite direction from what eAB gives.

Here is another possibility: instead of remaining entirely as kinetic energy in the system, suppose that the kinetic energy of the returning plate goes half into random kinetic, half into potential energy, as would be the case, say, if the energy were absorbed by a classical spring-mass lattice. Then the extra contribution to $\Delta\varphi$ is only half as much as before, or $-4M^2\varepsilon_o Av_o^3 q/Q^3/\hbar$, and the total phase shift is

$$\Delta\varphi = +(8/3)M^2\varepsilon_o Av_o^3 q/Q^3\hbar + (-4M^2\varepsilon_o Av_o^3 q/Q^3/\hbar) = -(4/3)M^2\varepsilon_o Av_o^3 q/Q^3\hbar. \tag{15}$$

Here is a third possibility: Suppose the kinetic energy of the returning plate is absorbed entirely as potential energy. Then K goes to zero and there is no extra contribution to $\Delta\varphi$, leaving us with our original result, the same as from eAB. Thus, the phase shift depends to a remarkable degree on the details of interactions in the overall system.

5 Discussion The accepted theory underlying eAB is incorrect because the wavefunction of the experimental apparatus cannot be ignored. Quantum interference occurs between different paths of the entire system in configuration space, and it is the Lagrangian of the entire system which determines the phase along each path. In the idealized experiments described long ago by Aharonov and Bohm, the effect of the electron on the rest of the system turns out to play an essential role. Why has this been so long overlooked? For two reasons, perhaps.

First, the vector and scalar potentials being components of a Lorentz-covariant four-vector, the individual contributions of **A** and V to the particle phase in eq. 2 are frame-dependent. With the magnetic Aharonov-Bohm effect well established in terms of a single-particle wavefunction, our relativistic instincts resist the thought that something more is required than to extend the single-particle Hamiltonian to include V, which leads directly to eAB.

Second, while the subject of quantum entanglement has been extensively explored in recent years, it seems most physicists have been unwilling to take seriously the existence of macroscopic quantum superpositions. In the matter at hand, the result stems precisely from entanglement of a particle with an experimental apparatus—that is, from a quantum superposition involving different states of a macroscopic system. Aharonov and Bohm recognized this possibility in 1961 but mistakenly concluded that the single-particle wavefunction nevertheless suffices.

An interference effect similar to eAB might be realized, but the result would depend critically (and perhaps unexpectedly) on the details of interactions within the experimental apparatus. The effect does not arise from a change in the electrical potential energy of the electron, which is cancelled by an equal and opposite change in the energy of the rest of the system. It is the Lagrangian of the entire system that determines the rate of phase generation, and while the electron itself is not subject to a net electric force, the electric forces it exerts on the source charges are responsible for the shift in the interference pattern.

Acknowledgments I would like to thank Kevin Martin for valuable comments and discussions, Mark Semon for alerting me to Aharonov and Bohm's 1961 paper, and Sharon Bertsch for technical assistance in preparing the manuscript.

References

1. Aharonov, Y., Bohm, D.: Significance of electromagnetic potentials in the quantum theory. Phys. Rev. **115**, 485-489 (1959)
2. Chambers, R. G.: Shift of an electron interference pattern by enclosed magnetic flux. Phys. Rev. Lett. **5**, 3-5 (1960)
3. Tonomura, A., Osakabe, N., Matsuda, T., Kawasaki, T.: Evidence for Aharonov-Bohm effect with magnetic field completely shielded from electron wave. Phys. Rev. Lett. **56**, 792-795 (1986)
4. Sakurai, J. J.: Modern Quantum Mechanics (Revised Edition). Addison Wesley, New York (1993)
5. Van Oudenaarden, A., Devoret, M. H., Nazarov, Yu. V., Mooij, J. E.: Magneto-electric Aharonov-Bohm effect in metal rings. Nature **391**, 768-770 (1998)
6. Aharonov, Y., Bohm, D.: Further considerations on electromagnetic potentials in the quantum theory. Phys. Rev. **123**, 1511-1524 (1961)

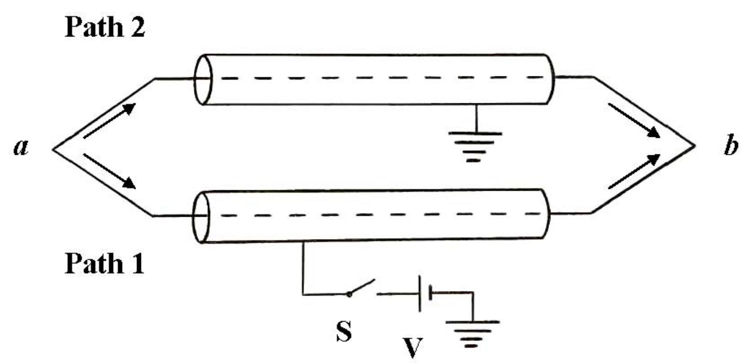

Fig. 1 Aharonov and Bohm's original idealized experiment, showing two interfering paths of an electron from *a* to *b*. The electric potential along path 1 is changed from 0 to V and back to 0 while the wavepacket is within the cylinders.

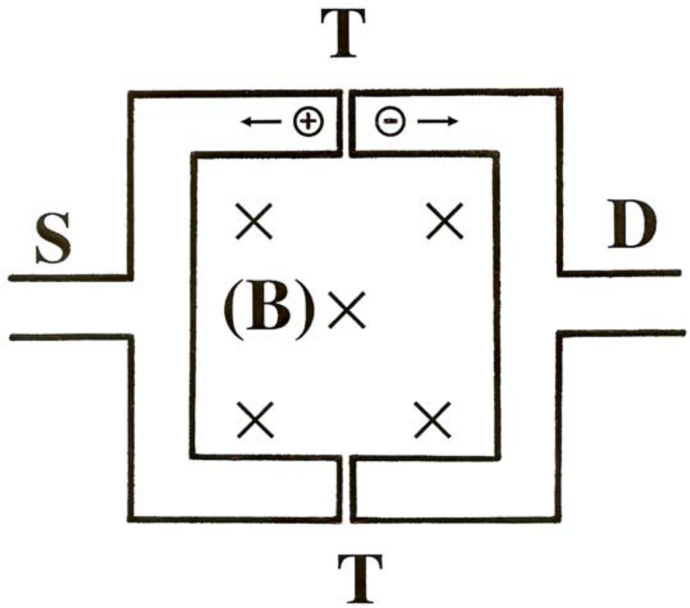

Fig. 2 Interference in a mesoscopic metal ring. Conductance is measured between source (S) and drain (D). An electron-hole pair formed at one tunnel junction (T) may recombine at the other junction, or the particles may proceed to the drain and source.

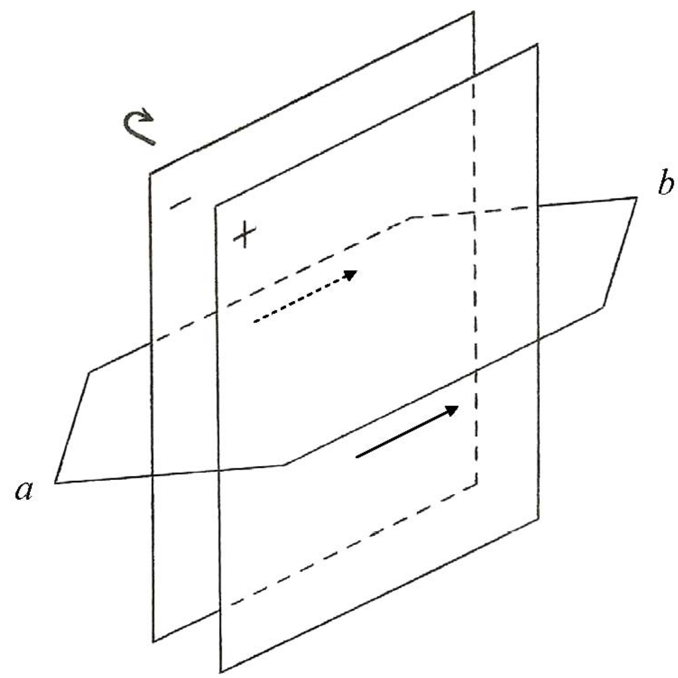

Fig. 3 Aharonov and Bohm's idealized experiment of 1961. The two interfering particle paths from *a* to *b* traverse opposite sides of a charged capacitor.